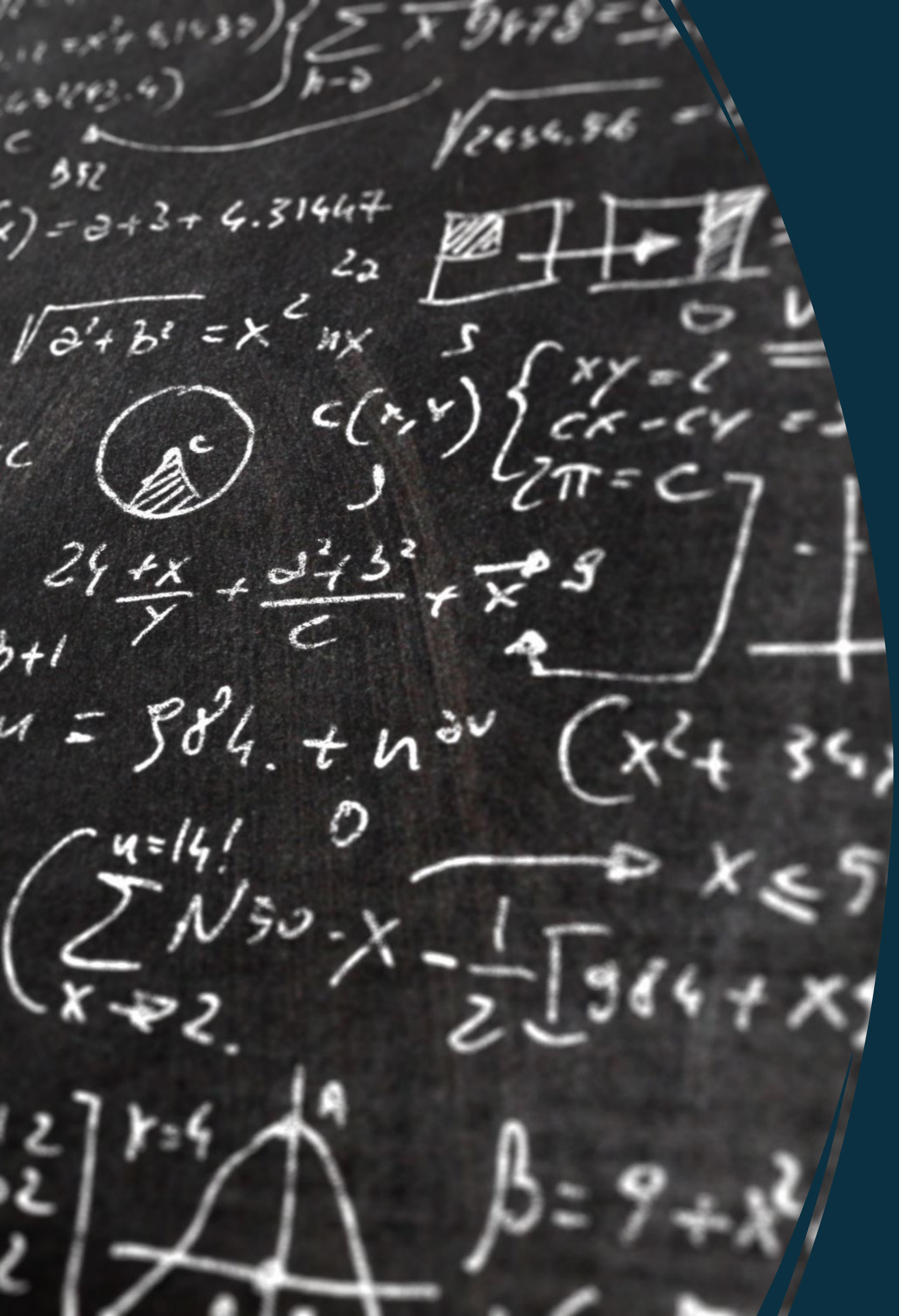

Disentangling Suboptimal Updating: Task Difficulty, Structure, and Sequencing

Author: Marina Agranov & Pellumb Reshidi

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Presenter: Andy

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Challenges to Bayesian Updating: Understanding Consistent Deviations

- In many situations, the standard **Bayesian updating framework accurately describes the evolution of beliefs** (Grether, 1978; Camerer, 1987; Charness and Levine, 2005)
- At the same time, there are **frequent and consistent deviations from Bayesian updating**.
  - These deviations persist even when people have ample opportunities to learn (Esponda et al., 2023) and
  - when stakes are very high (Gneezy et al., 2023)

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  - when stakes are very high (Gneezy et al., 2023)

Why?

Reason 1: The Inconsistent Behaviour Could Comes From Task Difficulty and Computational Complexity

- Different Definitions of Difficulty and Complexity:

  1. **Number of States and transitions required to implement a rule (Oprea, 2020; Banovetz and Oprea, 2022; Camara, 2021)**
  2. **Size of the state space (Ba et al. 2023)**
  3. **Number of distinct outcomes in the lottery support (Bernheim and Sprenger, 2020; Puri, 2022; Fudenberg and Puri, 2022)**
  4. The cognitive difficulty of aggregating outcomes and objective probabilities into a single value (Oprea, 2022)
  5. excess similarity between lotteries in the choice set (Enke and Shubatt, 2023)
  6. difficulty of evaluating streams of future payments (Enke et al., 2023)
  7. Cognitive Uncertainty (Enke and Graeber, 2023)

  …

  n. Signal Accuracy & Non-linearity of posterior updating – This paper proposed

Reason 2: Two Common Biases hinders Rational Belief Updating

- Recency Bias: People (irrationally) over-weight the most recent signal (Pitz and Reinhold, 1968; Edenborough, 1975; Grether, 1992)

- (Perfect) Base Rate Neglect: People tend to neglect the informative prior and always use uninformative prior as a starting point (Kahneman and Tversky, 1972; Bar-Hillel,1980)

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Research Question

1. How does **signal accuracy** plays a role in task **difficulty**?
2. Is the **signal accuracy** affect task difficulty by **non-linearity** of Bayesian updating?
3. Does **information structure** affect people's ability to perform Bayesian updating?
4. How does common biases (**Base Rate Neglect and Recency Effect**) in belief updating affect people's belief updating accuracy?

Instruction

Details of the Main Question

The experiment consists of several rounds.

In each round, a project will be selected randomly from a pool of projects (with each project having the same probability of being selected).

Within this pool of projects, **85%** of projects are Failures while **15%** are Successful.

Your task is to evaluate the chance that the project that was randomly selected is a Failure vs. Success.

To aid your evaluation, the computer will run a test on the selected project.

Test Accuracy is **80%** which means that:

- If the project is a Success the signal will be Positive with 80% probability and Negative with 20% probability.
- If the project is a Failure the signal will be Negative with 80% probability and Positive with 20% probability.

We will ask you to submit two evaluations:

- If the test is Positive, what is the chance that the project is a Success vs. Failure?
- If the test is Negative, what is the chance that the project is a Success vs. Failure?

Revision of Bayes Rule

$$\underbrace{\boxed{P(A|B)}}_{\text{posterior}} = \underbrace{\boxed{P(A)}}_{\text{prior}} \times \frac{\boxed{P(B|A)}\ \text{likelihood}}{\boxed{P(B)}\ \text{marginal}}$$

$$P(Failure \mid signal = positive) = P(Failure) * \frac{P(signal = positive \mid Failure)}{P(signal = positive)}$$

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In each round, a project will be selected randomly from a pool of projects (with each project having the same probability of being selected).

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Prior

Your task is to evaluate the chance that the project that was randomly selected is a Failure vs. Success.

To aid your evaluation, the computer will run a test on the selected project.

Test Accuracy is **80%** which means that:

Likelihood

- If the project is a Success the signal will be Positive with 80% probability and Negative with 20% probability.
- If the project is a Failure the signal will be Negative with 80% probability and Positive with 20% probability.

We will ask you to submit two evaluations:

- If the test is Positive, what is the chance that the project is a Success vs. Failure?
- If the test is Negative, what is the chance that the project is a Success vs. Failure?

Questions

Note: Marginal not directly given but can be calculated

Feedback Afterwards

The Test was Negative.

The Project was a Failure.

Previous Rounds' Outcomes

Round	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Signal	N	P	N	N	P	P	P	N	N	N	P	P	N	N	N	P
Project	F	F	F	F	F	S	S	F	F	F	S	S	F	F	S	S

P(Positive), N(Negative), S(Success), F(Failure)

12 Treatments: can be classified into 4 categories to investigate Information Structure

Uninformative Prior: If the states are randomly selected initially. All prior probability $p_i = \frac{1}{state\ space}$

1. **Baseline**: Informative Prior + 1 Signal with accuracy θ_2
2. **Simultaneous** : Uninformative Prior + 2 Signals Simultaneously with accuracy $\theta_1 = p_0\ and\ \theta_2$
3. **Sequential High-Low**: Uninformative prior + two Signals Sequentially with higher accuracy signal first
4. **Sequential Low-High**: Uninformative prior + two Signals Sequentially with lower accuracy signal first

Baseline and Others - **Equivalent mathematically** but different in presentation (Info Structure)

$$P(Failure \mid signal\ =\ positive)\ = P(Failure) * \frac{P(signal = positive \mid Failure)}{P(signal = positive)}$$

1. **Baseline:** *Informative Prior* $(P(F) = p_0\ and\ P(S) = 1 - p_0)$ + 1 Signal with accuracy θ_2

$$\frac{P(F|s)}{P(S|s)} = \frac{P(s|F)}{P(s|S)}\frac{P(F)}{P(S)} = \frac{P(s|F)}{P(s|S)}\frac{p_0}{1-p_0},$$

Baseline and Others - **Equivalent mathematically** but different in presentation (Info Structure and sequence)

1. **Baseline:** *Informative Prior* $(P(F) = p_0 \; and \; P(S) = 1 - p_0)$ + 1 Signal with accuracy θ_2

$$\frac{P(F|s)}{P(S|s)} = \frac{P(s|F)}{P(s|S)}\frac{P(F)}{P(S)} = \frac{P(s|F)}{P(s|S)}\frac{p_0}{1-p_0},$$

2. **Simultaneous** : Uninformative Prior $P(F) = P(S) = 1/2$ + 2 Signals Simultaneously with accuracy $\theta_1 = p_0 \; and \; \theta_2$

$$\frac{P(F|s_2, s_1 = n)}{P(S|s_2, s_1 = n)} = \frac{P(s_2|F)}{P(s_2|S)}\frac{P(s_1 = n|F)}{P(s_1 = n|S)}\frac{P(F)}{P(S)} = \frac{P(s|F)}{P(s|S)}\frac{\theta_1}{1-\theta_1}\frac{\tilde{p}_0}{1-\tilde{p}_0}. \quad = \frac{P(s|F)}{P(s|S)}\frac{p_0}{1-p_0}\frac{1/2}{1/2}.$$

Baseline: Informative Prior + 1 Signal

Probability Evaluation (Round 1)

Prior Information:

- 85% of Projects are Failures; 15% of Projects are Successful.
- Test Accuracy is 80% which means that:
  - If the project is a Success the signal will be Positive with 80% probability and Negative with 20% probability.
  - If the project is a Failure the signal will be Negative with 80% probability and Positive with 20% probability.

If the test is **Positive**
what is the chance that the project is a Failure vs Success?

Failure 28% **72% Success**

If the test is **Negative**
what is the chance that the project is a Failure vs Success?

Failure 93% **7% Success**

Submit Evaluation

Simultaneous: Uninformative Prior +2 Signals

Probability Evaluation (Round 1)

Prior Information:

- 50% of Projects are Failures; 50% of Projects are Successful.
- Test 1 Accuracy is 85% which means that:
  - If the project is a Success the signal will be Positive with 85% probability and Negative with 15% probability.
  - If the project is a Failure the signal will be Negative with 85% probability and Positive with 15% probability.
- Test 2 Accuracy is 80% which means that:
  - If the project is a Success the signal will be Positive with 80% probability and Negative with 20% probability.
  - If the project is a Failure the signal will be Negative with 80% probability and Positive with 20% probability.

If **Test 1** is **Negative**
and **Test 2** is **Positive**
what is the chance that the project is a Failure vs Sucess?

Failure % —— % **Success**

Misaligned Signal

If **Test 1** is **Negative**
and **Test 2** is **Negative**
what is the chance that the project is a Failure vs Sucess?

Failure % —— % **Success**

Aligned Signal

Sequential: Uninformative Prior +2 Signals

Probability Evaluation (Round 16)

Prior Information:

- 50% of Projects are Failures; 50% of Projects are Successful.
- Test 1 Accuracy is 85% which means that:
  - If the project is a Success the signal will be Positive with 85% probability and Negative with 15% probability.
  - If the project is a Failure the signal will be Negative with 85% probability and Positive with 15% probability.
- Test 2 Accuracy is 80% which means that:
  - If the project is a Success the signal will be Positive with 80% probability and Negative with 20% probability.
  - If the project is a Failure the signal will be Negative with 80% probability and Positive with 20% probability.

Test 1 is Positive.

Misaligned Signal

If Test 2 is **Positive**
what is the chance that the project is a Failure vs Success?

Failure **6%** **94% Success**

If Test 2 is **Negative**
what is the chance that the project is a Failure vs Success?

Failure **44%** **56% Success**

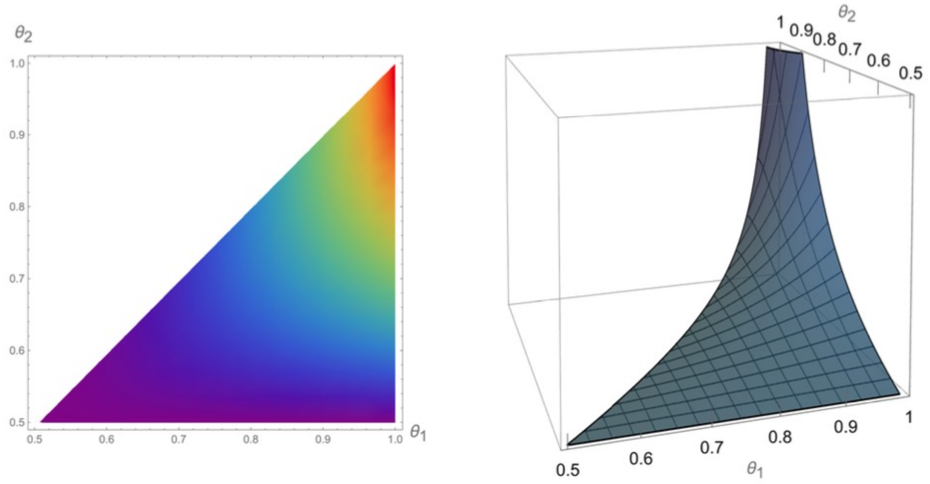
Vary Signal Accuracy to Manipulate Difficulty – It is due to non-linearity in posteriors

E.g. two signals are drawn s1 = n and s2 = p; compare the two cases:

1. $\theta 1 = 0.85$ and $\theta 2 = 0.80$; P(Success)? **41.3%**
2. $\theta 1 = 0.97$ and $\theta 2 = 0.92$; P(Success)? **26.2%**

Vary Signal Accuracy to Manipulate Difficulty – It is due to non-linearity in posteriors

Figure 1: Bayesian Posterior Derivative w.r.t high accuracy signal, θ_1



Notes: The left graph depicts a heatmap (warmer colors represent higher values), and the right graph is 3D plot. In both graphs, we focus on $\theta_1 \geq \theta_2$, since θ_1 denotes the high accuracy signal.

- The higher signal accuracy, the more non-linear the function is
- The higher accuracy gap between two signals, the more non-linear the function is

Vary Signal Accuracy to Manipulate Difficulty – It is due to non-linearity in posteriors

Table 1: Sessions, Treatments, and Parameter Values

Session	# Participants	Treatment	Parameter	Prior	Low Accuracy Signal	High Accuracy Signal
1	101	Baseline	$\tilde{A}$	p=0.85	$\theta_2 = 0.80$	—
2	101	Simultaneous	A	p=0.50		$\theta_1 = 0.85$
3	101	Sequential High-Low				
4	100	Sequential Low-High				
5	99	Baseline	$\tilde{B}$	p=0.95	$\theta_2 = 0.85$	—
6	99	Simultaneous	B	p=0.50		$\theta_1 = 0.95$
7	102	Sequential High-Low				
8	100	Sequential Low-High				
9	99	Simultaneous	C	p=0.50	$\theta_2 = 0.75$	$\theta_1 = 0.85$
10	100		D		$\theta_2 = 0.80$	$\theta_1 = 0.90$
11	100		E		$\theta_2 = 0.85$	$\theta_1 = 0.90$
12	100		F		$\theta_2 = 0.90$	$\theta_1 = 0.95$

Easiest

Hardest

In Btw

Participants

- 1202 participants (100 per treatment) from Prolific in 2022
  - Offloading? What if people open Bayesian calculator online?
- Between 18-70 years old US participants
- Fluent in English
- Sex balanced

Incentives

$5 fixed payment + 20% chance to be selected into a bonus group.

- For the selected participants, one of the experimental rounds was randomly chosen for payment.
- The answers submitted in the chosen round determined whether the selected participant received an additional bonus of $20 through a "standard" BDM mechanism
  - Actually, not standard and very complicated

BDM Mechanism (Quick Showcase)

How Payments are Calculated

In every question of this type, you will use the slider to indicate the probabilities of Failure and Success.

Let X represent your chosen probability of Failure, and consequently 100 - X will be your chosen probability of Success.

After you submit your choice of X, the program will generate a number from 0 to 100, with each number being equally likely. Call this number Y. Your chosen number X, the randomly generated number Y, and whether the outcome is Failure or Success will determine your chances of winning $20. If Y is greater than or equal to X, you will win $20 with Y% chance. If Y is less than X, you will win $20 if the outcome is Failure.

Given this payment scheme, it is always in your best interest to choose X that represents your best evaluation of the chance that Failure and Success will happen.

The important thing to remember is that we have chosen the payment scheme so that it is always in your best interest to honestly report your best evaluation of the chance that Failure and Success happens.

Denote $f \in [0,1]$ is the probability that a project fail

Denote $x \in [0,1]$ is the participant's true belief of this project is going to fail

Denote $\hat{x} \in [0,1]$ as participant's reported belief of this project is going to fail

Denote $y \in [0,1]$ as a randomly generated number

The payment structure (BDM) is

$$payoff = \begin{cases} \$20 * y & if\ y \geq \hat{x} \\ \$20 * f & if\ y < \hat{x} \end{cases}$$

The idea is it is the participant's best interest to always report $\hat{x} = x$

Proof:

If report $\hat{x} > x$, there are three possibilities.

1. $y \geq \hat{x} > x$. In this case, payoff will be $\$20 * y$. It is the same if we report x
2. $\hat{x} > y > x$. In this case, payoff will be $\$20 * f$. Payoff will be $\$20 * y$ if we report x. If participant believe $f = x$, participant better off report x (Payoff $\$20 * y > \$20 * x$)
3. $\hat{x} > x > y$. In this case, payoff will always be $\$20 * f$. It is the same if we report x

If report $\hat{x} < x$, there are three possibilities.

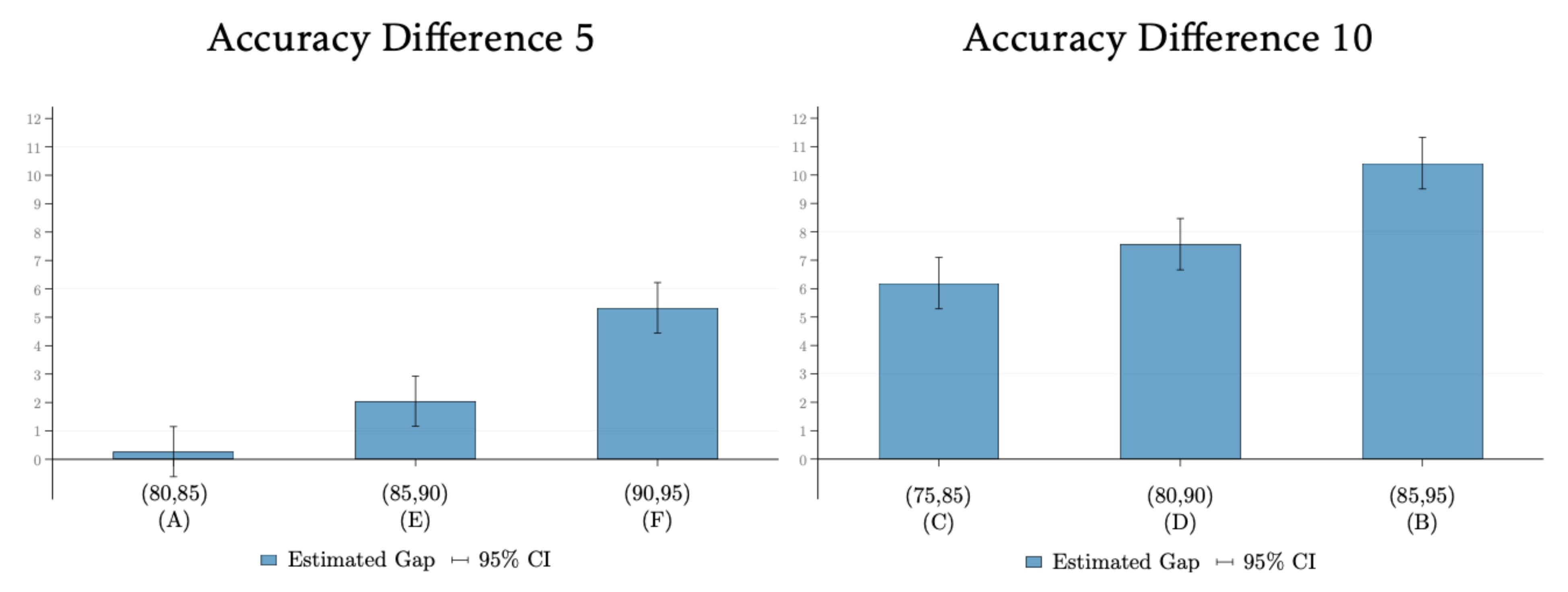
4. $y < \hat{x} < x$. In this case, payoff will be $\$20 * f$. It is the same if we report x
5. $\hat{x} \leq y < x$. In this case, payoff will be $\$20 * y$. Payoff will be $\$20 * f$ if we report x. If participant believe $f = x$, participant better off report x (Payoff $\$20 * x > \$20 * y$)
6. $\hat{x} < x < y$. In this case, payoff will always be $\$20 * y$. It is the same if we report x

Therefore, report $\hat{x} = x$ weakly dominates other strategies.

The important thing to remember is that we have chosen the payment scheme so that it is always in your best interest to honestly report your best evaluation of the chance that Failure and Success happens.

Result 1: The level of and difference between signal accuracies increase the gap between observed and Bayesian posteriors - Simultaneous

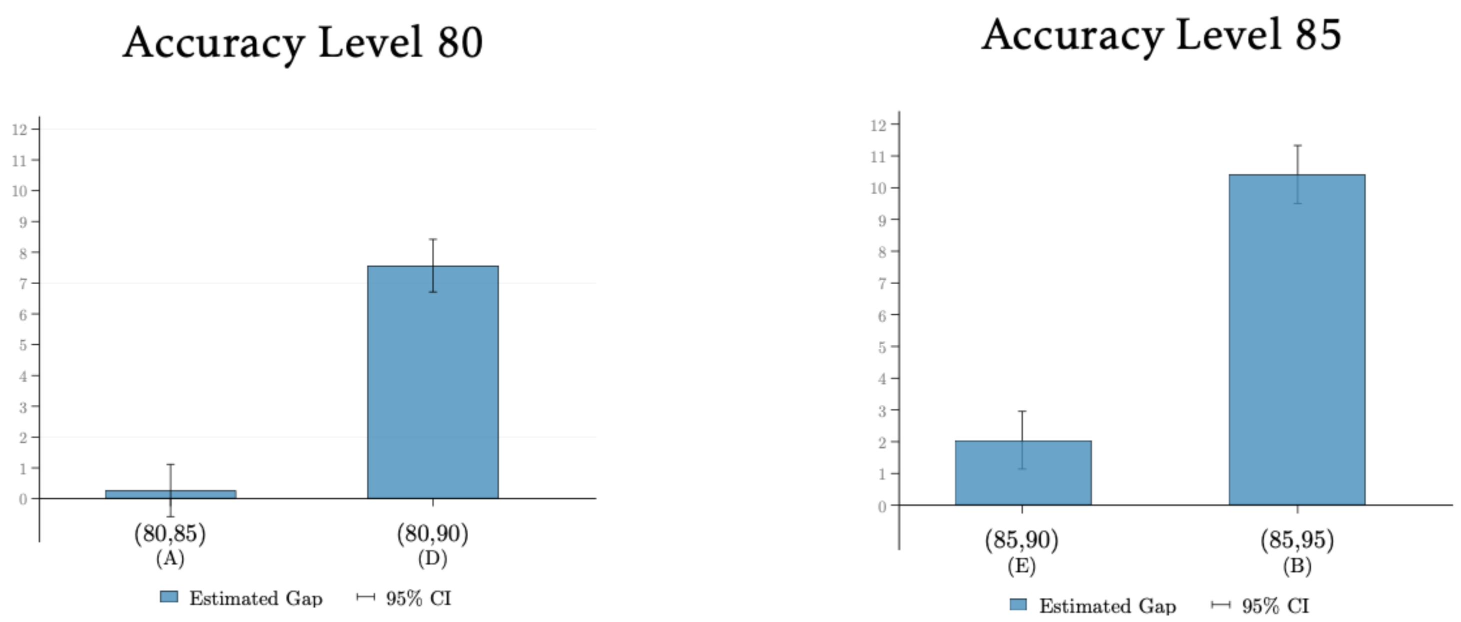
Figure 2: The Impact of Signal Accuracy Levels on Task Difficulty



Introduction | Method | Result | Discussion

Result 1: The level of and difference between signal accuracies increase the gap between observed and Bayesian posteriors - Simultaneous

Figure 3: The Impact of Signal Accuracy Difference on Task Difficulty



Introduction | Method | Result | Discussion

Result 1: The level of and difference between signal accuracies increase the gap between observed and Bayesian posteriors - Simultaneous

Table 2: Accuracy level and Difference impact on observed Gap

	Gap	
	All Rounds	Last 5 Rounds
Difference	1.566***	1.555***
	(0.240)	(0.289)
Level	0.464***	0.421***
	(0.124)	(0.152)
N (observations)	11980	2995
K (individuals)	599	599

Individual-level clustered errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Result 2: Participants are capable of incorporating nonlinearities only partially. Behavior is best described by a model that roughly lies between Bayesian and linear updating.

$$\tilde{\pi}(S|s_2 = p, s_1 = n) = \alpha \underbrace{\frac{\theta_2(1-\theta_1)}{\theta_2+\theta_1-2\theta_2\theta_1}}_{\text{Bayesian Posterior}} + (1-\alpha)\underbrace{\left(\frac{1}{2}-\theta_1+\theta_2\right)}_{\text{Fully Linear}}.$$

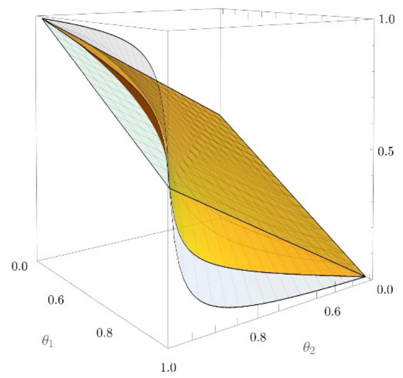
Table 3: Estimated α

	All Rounds	Last 5 Rounds
$\hat{\alpha}$	0.417***	0.564***
	(0.0555)	(0.0663)
N (observations)	11980	2995
K (individuals)	599	599

Individual-level clustered errors in parentheses

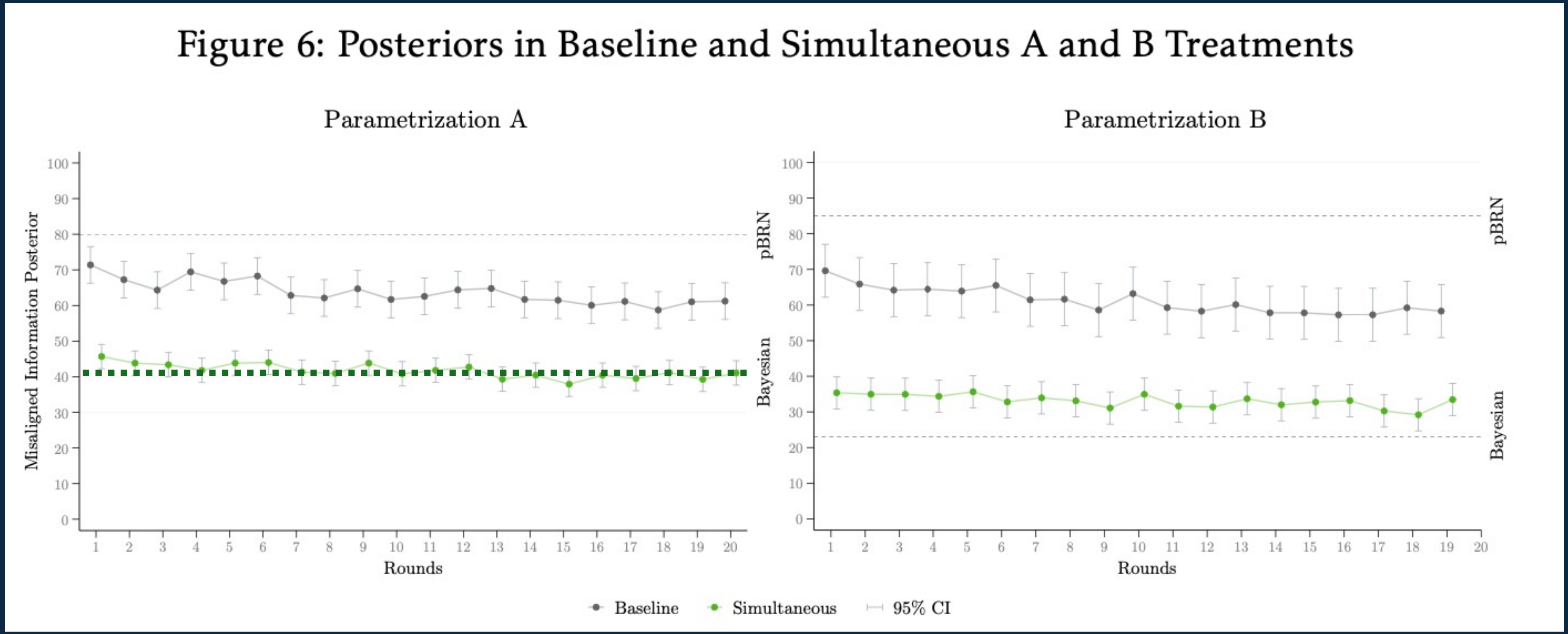
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Figure 4: Estimated Model

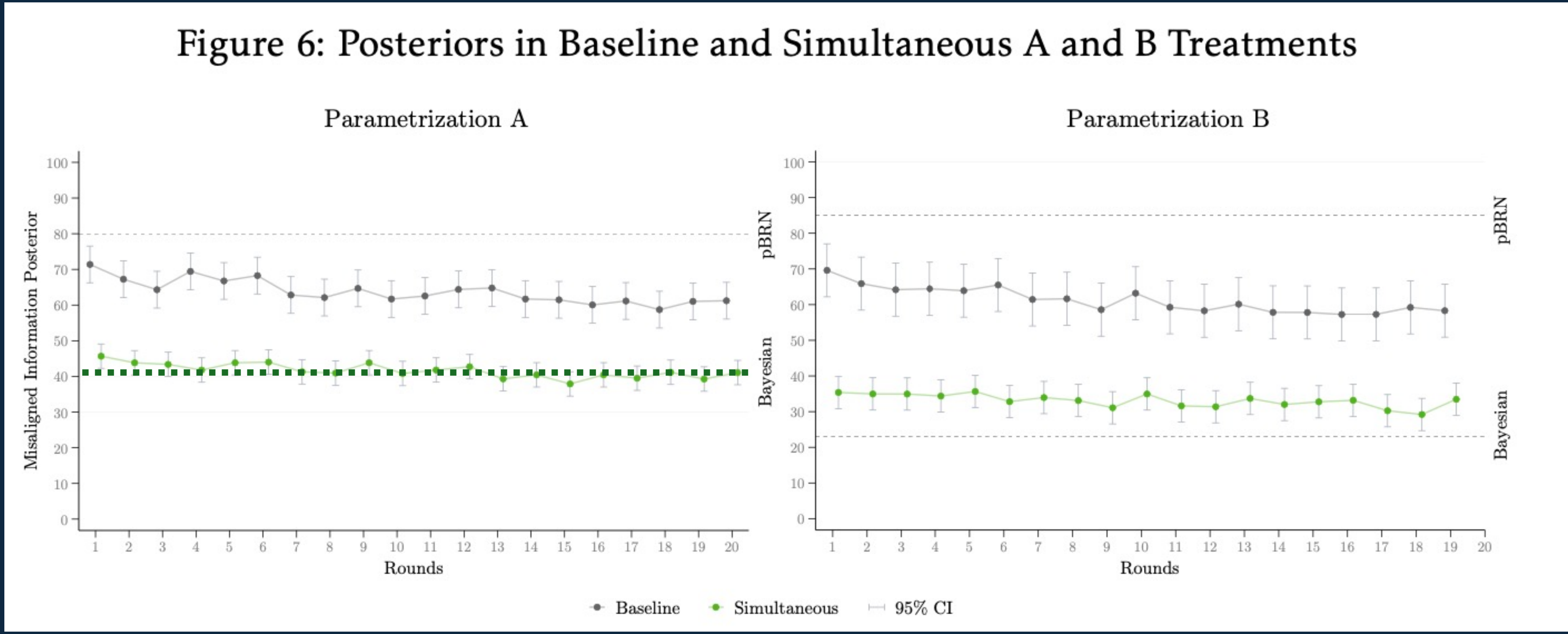


Notes: The figure illustrates the Bayesian ($\alpha = 1$) and fully linear ($\alpha = 0$) models through transparent graphs, along with the estimated model ($\alpha = 0.56$) via the yellow graph.

Result 3: In low-difficulty environments, delivering information through two simultaneous signals leads to an estimated mean statistically indistinguishable from the Bayesian posterior. - Simultaneous

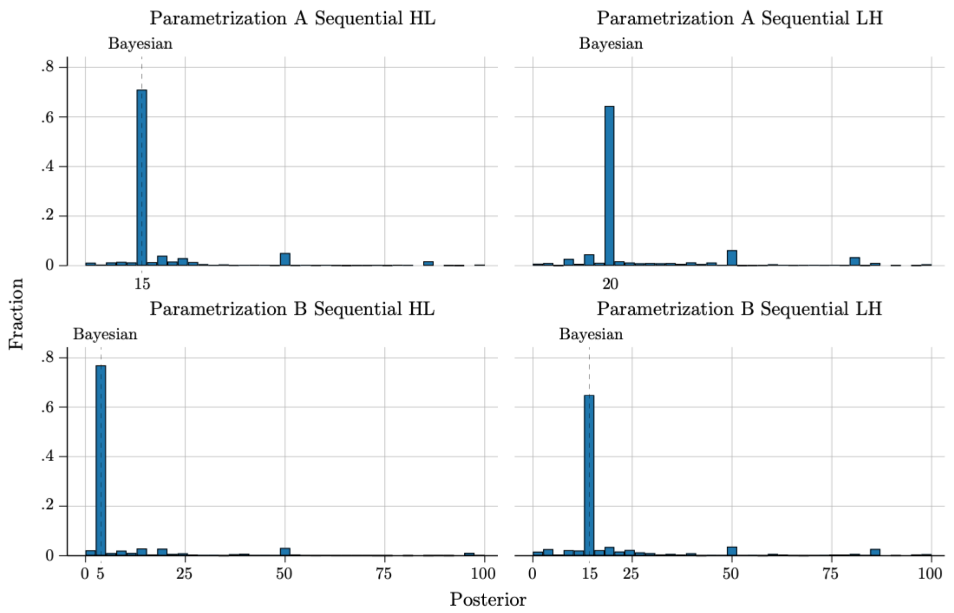


Result 4: In the high-difficulty environment, delivering information through two simultaneous signals decreases the gap between the observed and Bayesian beliefs but does not eliminate it.- Simultaneous



Result 5.1: In sequential HL treatment, the belief elicited after the first signal is indistinguishable from the baseline (informative) prior

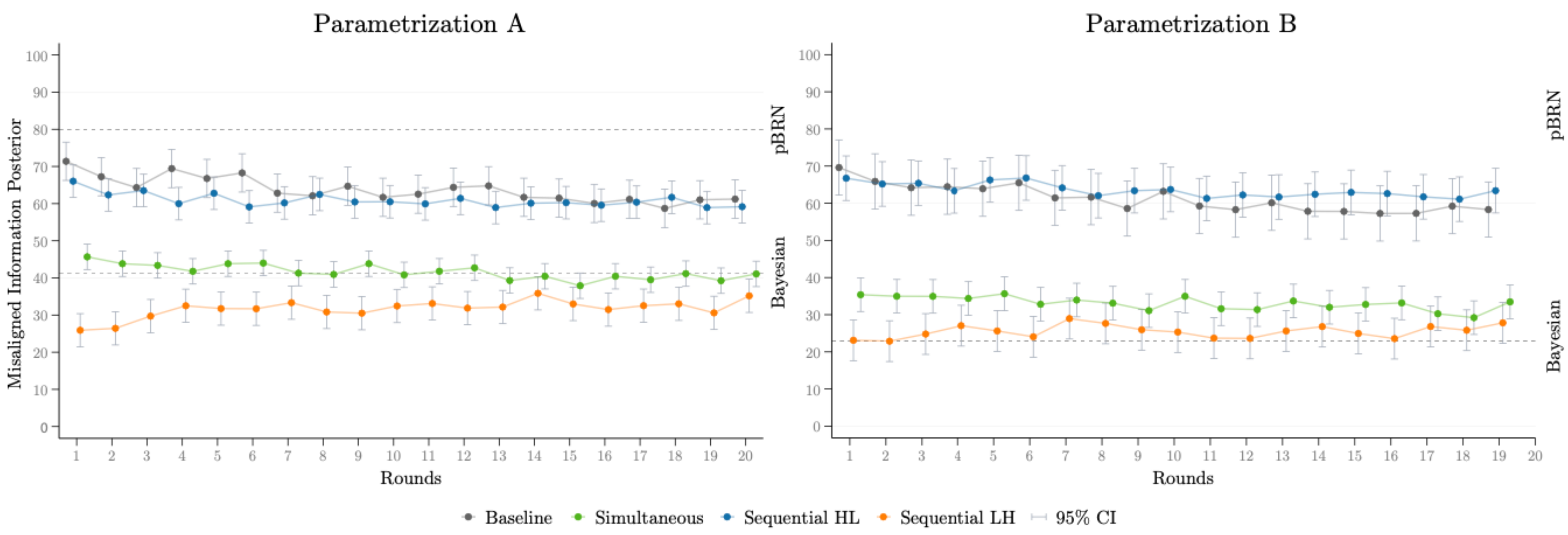
Figure 5: Sequential Treatments - Posteriors after the First Signal



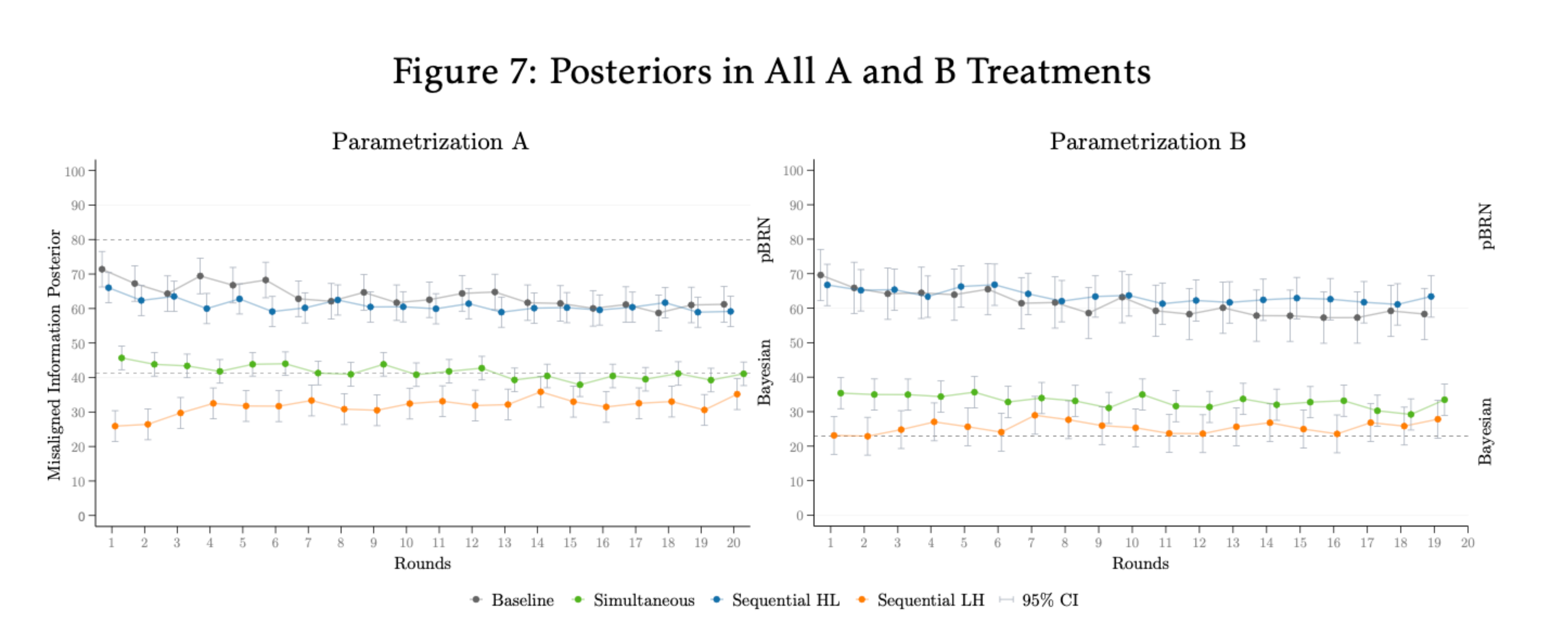
Notes: Above, we report the histogram of participants' posteriors. The vertical axis represents the fraction of choices, whereas the horizontal axis corresponds to the particular posterior.

Result 5.2: No difference in elicited beliefs at the aggregated level between sequential HL treatment and baseline

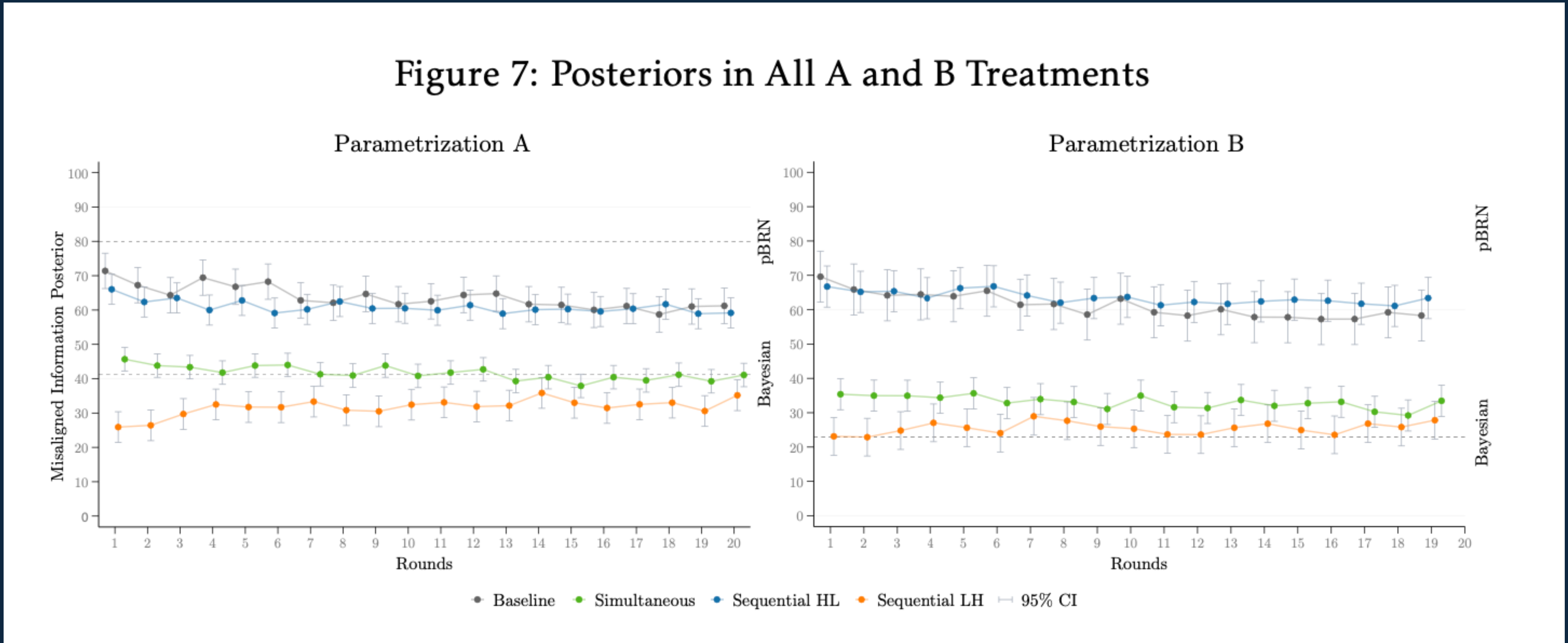
Figure 7: Posteriors in All A and B Treatments



Result 5.3: We document no aggregate effect on participants' reported beliefs when altering the information structure. – Not convinced

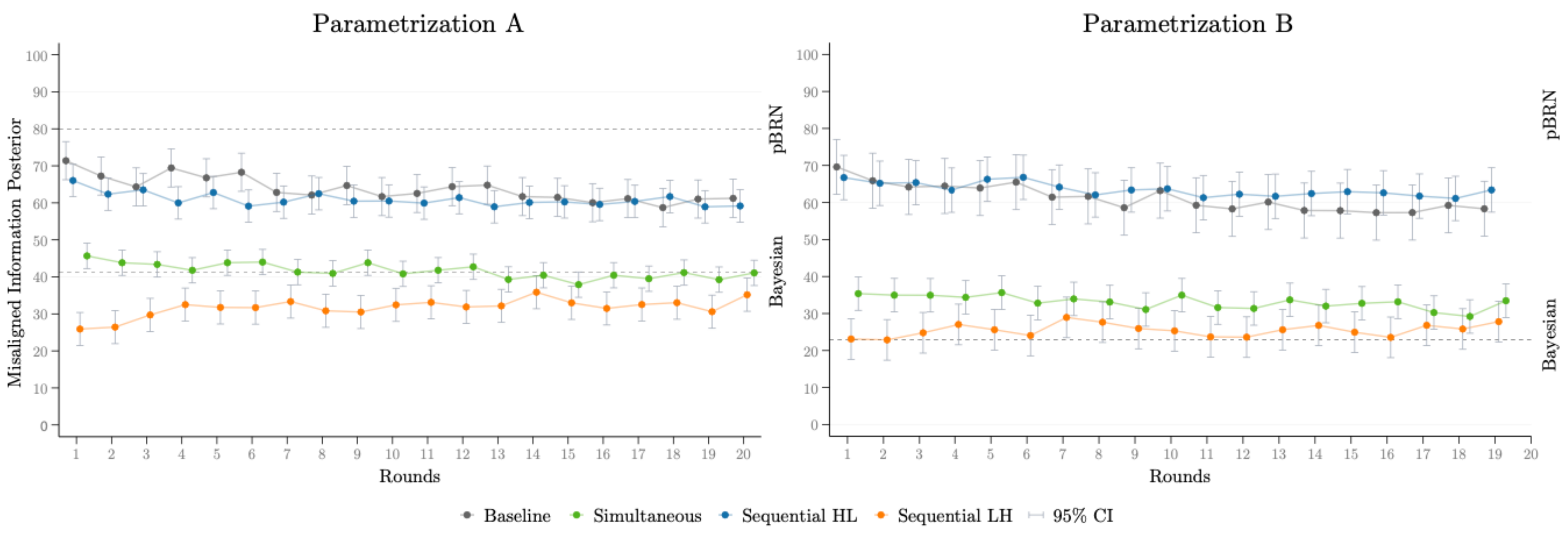


Result 6: We document a sizable recency bias independent of signal accuracy and task difficulty



Result 7: The ranking between all four treatments is preserved across different parameterizations, as are results regarding information structure and sequencing.

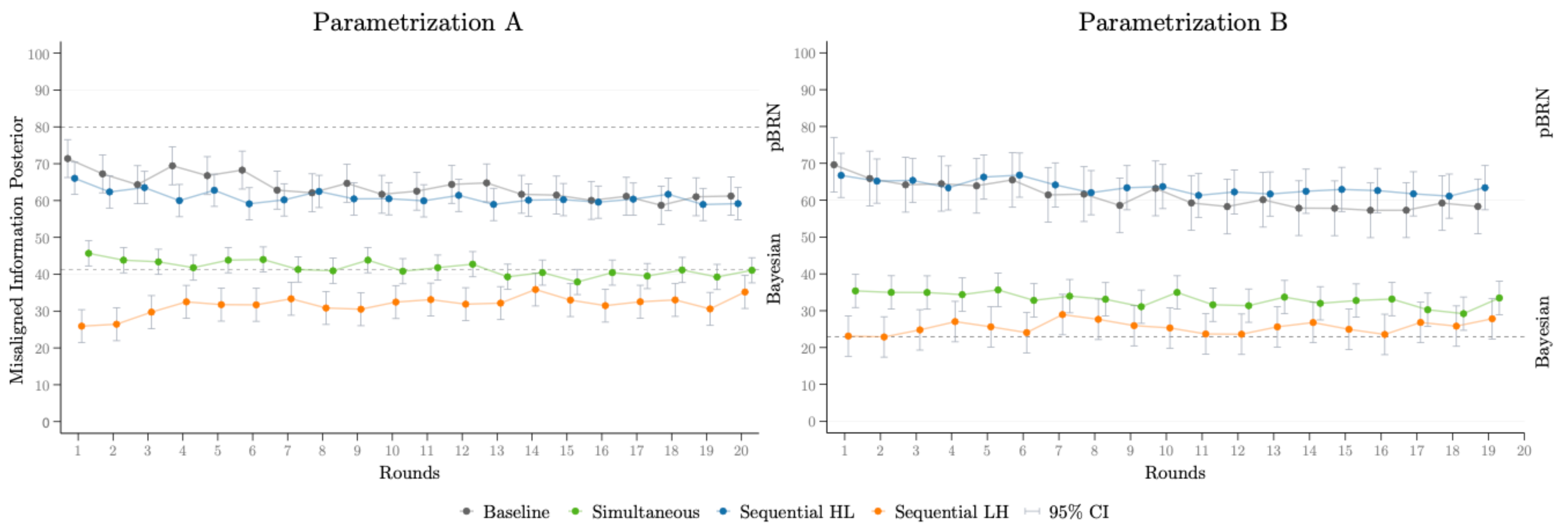
Figure 7: Posteriors in All A and B Treatments



Introduction | Method | Result | Discussion

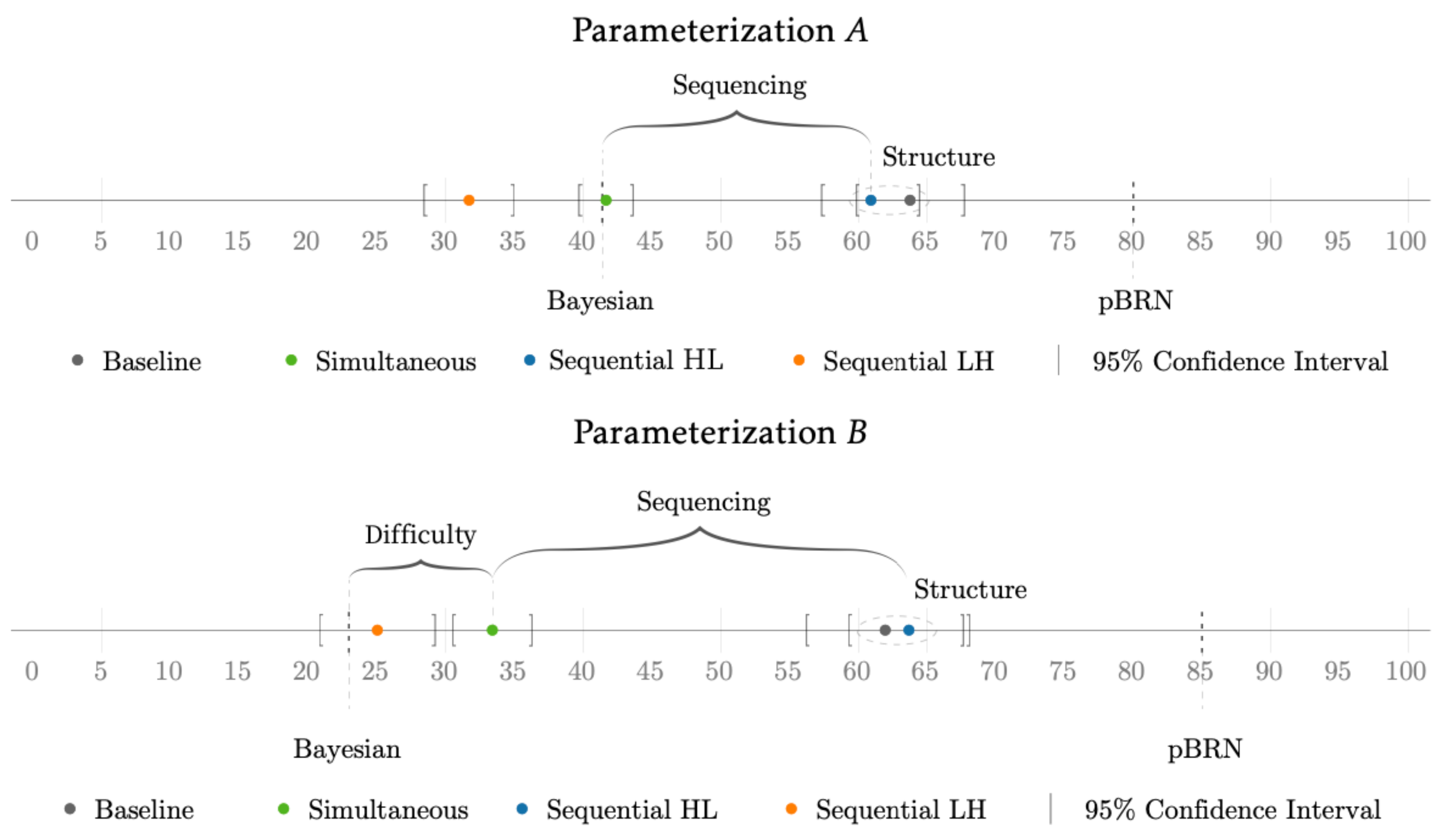
Result 8 (Countering Biases). The bias arising from sequential information arrival (recency bias) can help mitigate the difficulty bias, which arises from non-linear thinking required to reach the Bayesian posterior. - Not convinced

Figure 7: Posteriors in All A and B Treatments



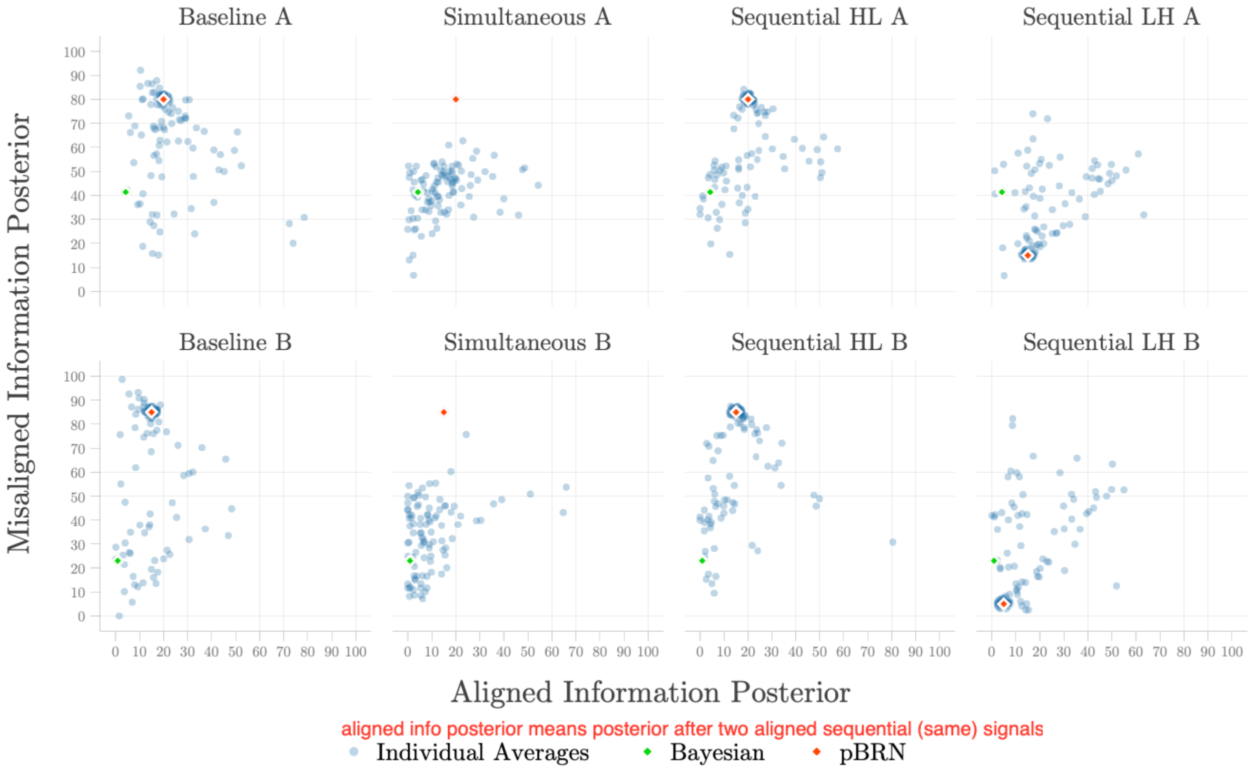
Result 9: Information sequencing is the main catalyst of base- rate neglect, with task difficulty also playing a significant role.

Figure 8: Average posteriors in all treatments



Result 10: The Simultaneous treatment is the only treatment leading to individual- level beliefs that are not strongly concentrated around the pBRN level.

Figure 9: Average Individual Choices



Notes: To help distinguish the large amount of data bundled on the pBRN level, we apply a jitter of 1.5 magnitude. This jittering perturbs the datapoint no further than a distance of 1.5 from the initial value. The top(bottom) row displays data across treatments under parametrization A(B).

pBRN means perfect base rate neglect

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pBRN means perfect base rate neglect

Result 11 (Effect of Information Structure). Elicited beliefs of participants who exclusively rely on recent information are unaffected by information structure. For other participants, a change in the information structure results in less extreme reported beliefs.

Figure 10: Distribution of Posteriors and Individual-Level Averaged Posteriors

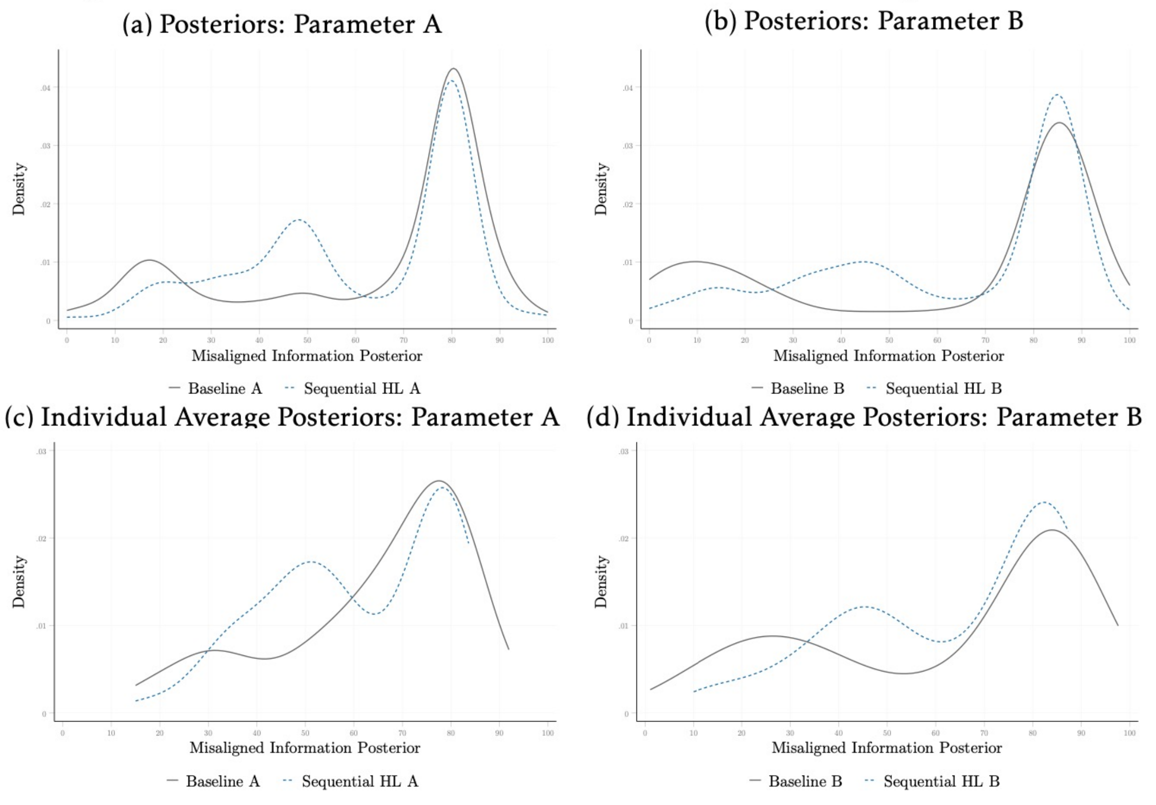
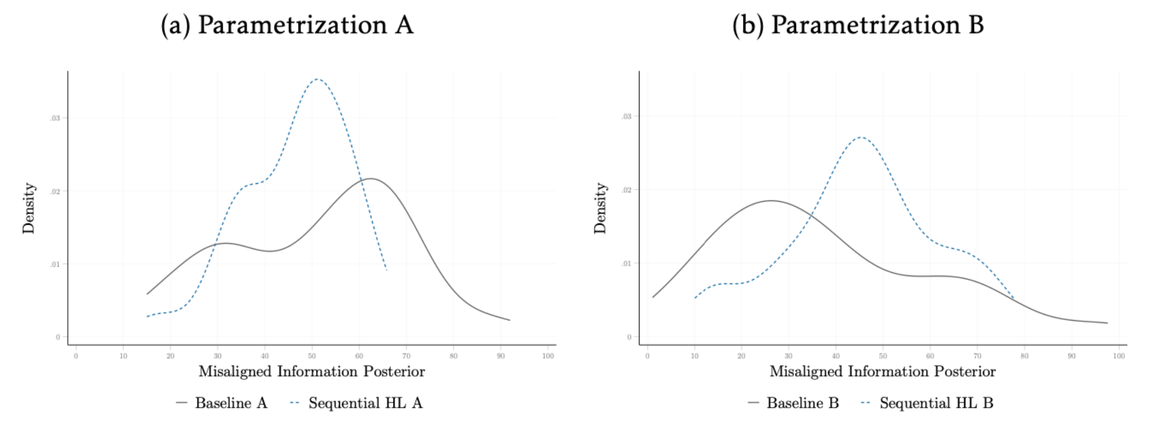
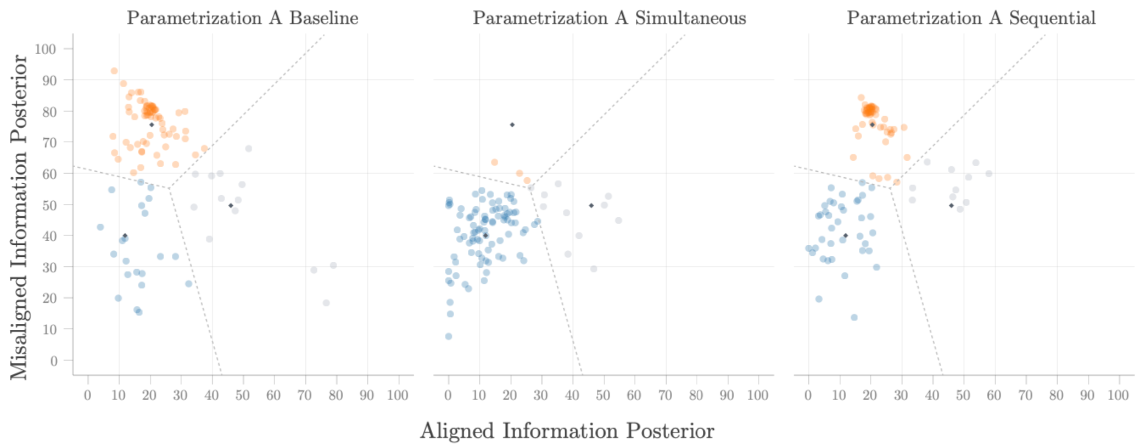


Figure 11: Individual-Level Averaged Posteriors Excluding pBNE



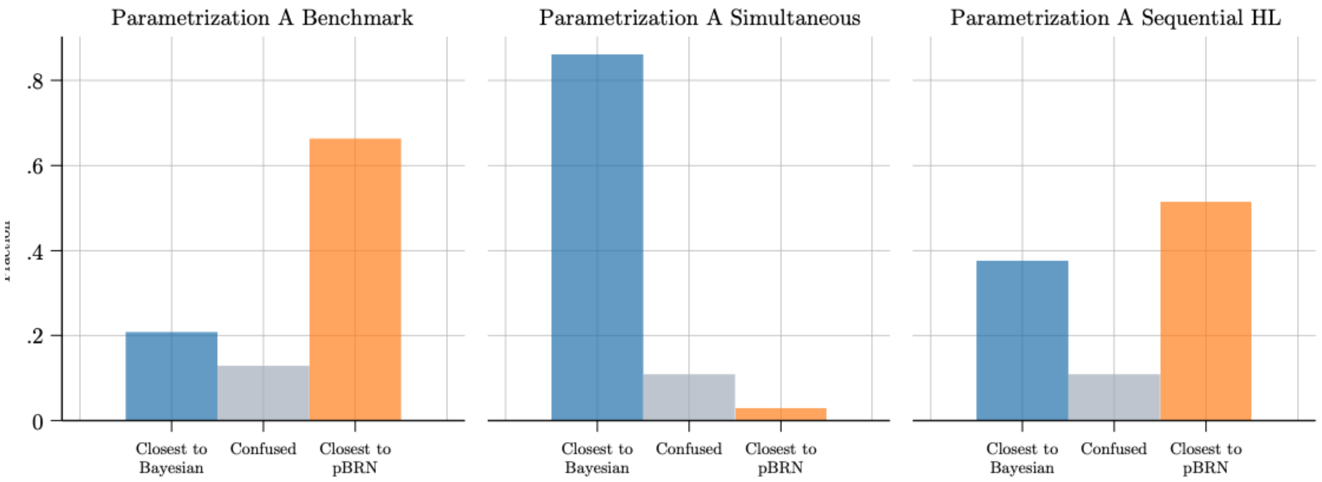
Result 12 Information structure affects participant categorization. – K-Means clustering, found centroid very close to Bayesian and pBRN in Easy Case

Figure 12: Parametrization A Clustering



Notes: Participants are categorized into three separate clusters. Dark gray dots mark the centroids of these clusters, and dashed lines represent the Voronoi cells corresponding to these centroids.

Figure 13: Parametrization A Cluster Histogram



Result 13: We observe variation in both clusters as well as the distribution of participants among these clusters across different parameters. – Hard Case K-Means Clustering

Figure 14: Parametrization B Clustering

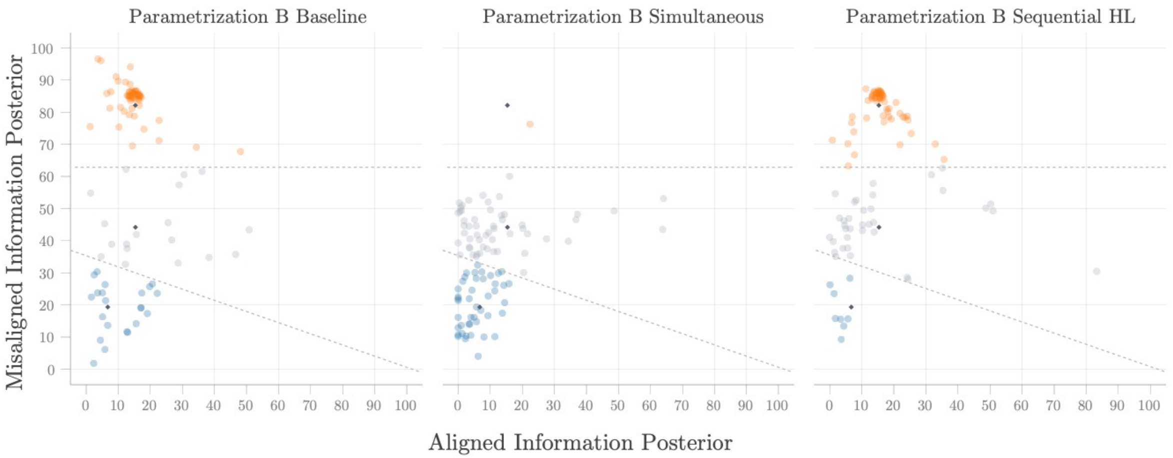


Figure 15: Parametrization B Cluster Histogram